Handout Lecture 7: Conditionals

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Abstract. This handout is devoted to the logic of counterfactual conditionals. The focus is on the the possible worlds similarity semantics due to Lewis [6].

1 Introduction

A counterfactual conditional is a statement of the form "if p, then q" where the antecedent (the *if* clause) p is known to be false. Typical examples are:

(1) a. If the electricity had not failed, dinner would have been ready on time
b. If the door were open, I would get the key I left in there
c. If you had looked into your pocket, you might have found a penny

Grammatically, in English at least, they are often signaled by the subjunctive mood in the antecedent and the auxiliary would/might in the consequent.

⚠ The term "counterfactual" was coined by Goodman [4] in reference to Chisholm [2] 's notion of a "contrary-to-fact conditional".

Counterfactual conditionals are often contrasted with so-called indicative conditionals:

(2) a. If Oswald did not kill Kennedy, someone else did
b. If Oswald had not kill Kennedy, someone else would have

Clearly the two conditionals differ in meaning. (2a) is an indicative: it signals that it is an open possibility that Oswald did not kill Kennedy, and will be judged true by anyone who knows that Kennedy was killed. (2b) is a counterfactual: it signals that it is taken for granted that Oswald did kill Kennedy, and makes the somewhat dubious claim that his assassination was inevitable.

However, we should be careful with suggestions from natural language. There are some formally indicative counditionals that express counterfactuality. And the "subjunctive" marking doesn't always mean counterfactuality. Examples:

(3) a. No Hitler, no A-bomb
b. If he has solved this problem, I'm the Queen of England
c. If Jones had taken arsenic, he would have shown just exactly the same symptoms which he actually shows.

The first two examples shows that the subjunctive marking is not necessary for counterfactuality. (3b) is an indicative. The third example shows that the subjunctive marking is not even sufficient for counterfactuality.

The study of counterfactual reasoning has given rise to an important literature, both in philosophy and in computer science. Ginsberg & Ginsberg [3] among others have stressed the importance of counterfactual reasoning in AI. This kind of reasoning plays a role in planning and diagnosis analysis (which requires identifying causes of observed effects among a set of possibilities). Halpern [5] has also argued that counterfactual reasoning plays an important role in analyzing rationality in games. In deciding what to do at a given time, a player must analyze what would have happened had he done something else.

There are close similarities between the logic of counterfactuaI conditionals, some versions of nonmonotonic or defeasible inference (as constructed in the 1980's in the context of logics for artificial intelligence), belief or theory revision (especially in the 1985 paradigm of Alchourrón, Gärdenfors and Makinson), updating (again, in artificial intelligence literature and especially in work of Katsuno and Mendelzon) and the logics of conditional obligation (as put forth by Hansson and van Fraassen in the early 1970's). The connection between these different areas is discussed in length by Makinson [7].

Notoriously, counterfactual conditionals cannot be represented as material conditionals, for the falsehood of the antecedent automatically makes any material conditional true, which is certainly not the case for these conditionals. Neither can they be represented as strict implications, because strict implication satisfies a number of laws that the counterfactual conditional does not, like the principle of transitivity (cf. section 2.3). Logicians have been working on the representation of counterfactual conditionals for several decades, and have developed a number of mathematical constructions to model them. This handout focuses on one of the most well-known approaches to counterfactuals, the possible worlds similarity approach due to Lewis [6]. It is itself closely related to a semantics first put forth by Stalnaker [9].

The basic idea underpinning the account is perhaps best explained with reference to Ramsey [8], who suggested the following procedure for evaluating an indicative conditional–this has become known as "Ramsey's test".

> "If two people are arguing 'if ϕ will ψ?' and both are in doubt as to ϕ, they are adding ϕ hypothetically to their stock of knowledge and arguing on that basis about ψ. " [8, p. 143]

Several strategies for extending Ramsey's test into a full-blooded theory of conditionals have been developed. The semantics described in this handout is one of them. It is based on Stalnaker's own interpretation of Ramsey's test in terms of minimal changes. Stalnaker is aware that this procedure is completely specified by Ramsey only in the case in which the agent has no opinion about the truth value of the antecedent of the conditional that is being evaluated. Therefore Stalnaker asks himself how the procedure suggested by Ramsey can be extended to cover counterfactuals. His answer is:

"According to [Ramsey's] suggestion, your deliberation [...] should consist of a simple thought experiment: add the antecedent (hypothetically) to your stock of knowledge (or beliefs), and then consider whether or not the consequent is true. Your belief about the conditional should be the same as your hypothetical belief, under this condition, about the consequent." [9, p. 102]

The proposed procedure for determining if a conditional holds has two steps. First, you update your stock of beliefs to accommodate for the truth of the antecedent. This is done by making as few changes as possible to your old beliefs. Next, you check whether the consequent is among your new beliefs.

Ramsey's test

A conditional is accepted if the consequent is true after we add the antecedent (hypothetically) to our stock of beliefs and make whatever minimal adjustments are required to maintain consistency.

2 The Lewis world-similarity semantics

2.1 Language

The language is generated by the following BNF:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid \phi \,\Box\!\!\rightarrow \phi$$

There is no prohibition against embedding counterfactual conditionals within counterfactual conditionals. These are called "nested counterfactuals". Example:

(4) a. If I had bought a Botticelli from John and if I had noticed afterwards it was a fake, I would have sued him

Other (modal) connectives are introduced by the abbreviations:

'Might'	$\phi \Diamond\!\!\rightarrow \psi$	is $\neg(\phi \,\Box\!\!\rightarrow \neg\psi)$
'Box'	$\Box\phi$	is $\neg\phi \,\Box\!\!\rightarrow \bot$
'Diamond'	$\Diamond\phi$	is $\neg(\phi \,\Box\!\!\rightarrow \bot)$

2.2 SOS semantics

In this section we introduce "system of spheres"(SOS) semantics put forth by Lewis [6].

Definition 1 (System of spheres, SOS)**.** *Let W be a non-empty set of possible worlds (or states). A system of spheres is an assignment \$ from W to a set of subsets of W, where for each $w \in W$,*[1]

- *$\$_w$ is centered on w: $\{w\} \in \$_w$;*
- *$\$_w$ is nested: for $S, T \in \$_w$, either $S \subseteq T$ or $T \subseteq S$;*

The members of $\$_w$ are called spheres around w. The first condition says that they are all "centered" around the base world w. The second condition tells us that the spheres are "nested", i.e., linearly ordered. For any two spheres, one is included in the other. Because of the third condition, $\cup\$_w$ is itself a sphere around w; it is the largest, or *outermost* sphere around w.

In a system of spheres, the worlds are intended to be ordered by their relative similarity to the base world w; hence the smaller a sphere is, the closer to w the worlds within that sphere are. Intuitively, the centering condition tells us that w is more similar to itself than it is to any other worlds. (See section 2.4).

[1] We write $\$_w$ for $\$(w)$.

More constraints can be introduced. For the purpose of this handout, we will not do it.

We may posit a two-place relation $\geq_w$ among worlds, regarded as the ordering of worlds in respect of their comparative similarity to w. This alternative modelling is discussed in more details in subsection 2.4

So-called Hamming distance is sometimes used as a means for measuring the degree of similarity amongs worlds, and hence for generating a system of spheres. We recall that the Hamming distance between two possible worlds is given by the number of propositional atoms on which they disagree.

Exercise 1. *Show that a system of spheres is closed under union and intersection.*

Exercise 2. *Show how to generate a system of spheres using the Hamming distance between possible worlds.*

Definition 2 (SOS model)**.** *A SOS model $M = (W, \$, V)$ is a tuple where W is a non-empty set of possible worlds, \$ is a system of spheres (as defined* supra*) and V is a valuation function.*

Definition 3 (Truth at a world)**.** *Given a SOS model $M = (W, \$, V)$ and a world $w \in W$, we define the relation $M, w \vDash \phi$ (reading: "world w satisfies ϕ in model M") by induction on ϕ using the following clauses:*

- *$M, w \vDash p$ iff $w \in V(p)$.*
- *$M, w \vDash \neg\phi$ iff $M, w \nvDash \phi$*
- *$M, w \vDash \phi \wedge \psi$ iff $M, w \vDash \phi$ and $M, w \vDash \psi$.*
- *$M, w \vDash \phi \,\Box\!\!\rightarrow \psi$ iff*
  a) $\neg\exists S \in \$_w \exists w' \in S \; M, w' \vDash \phi$, or
  b) $\exists S \in \$_w \exists w' \in S \, M, w' \vDash \phi \; \& \; (\forall w'' \in S)\,(M, w'' \vDash \phi \rightarrow \psi)$

Call a world that makes ϕ true a "ϕ-world", and a phere that contains at least one ϕ-world a ϕ-permitting sphere. The evaluation rule for the would-conditional makes $\phi \,\Box\!\!\rightarrow \psi$ true in two cases:

- a): no ϕ-permitting sphere (vacuous case);
- b): ψ holds at every ϕ-world in the smallest ϕ-permitting sphere (non-vacuous, principal case).

Intuitively, a) says that ϕ is impossible or counter-possible. Our intuitive gloss of b) presupposes the so-called *Limit Assumption*. It is the assumption that as we take smaller and smaller antecedent-permitting spheres we eventually reach a limit: the smallest antecedent-permitting sphere, and in it the closest antecedent-worlds. Let $[\phi]$ be the truth-set of ϕ in model M, viz. $\{w \in W : M, w \models \phi\}$. Formally, the limit assumption may be stated thus:

$$[\phi] \cap (\cup\$_w) \neq \emptyset \Rightarrow \cap\{S \in \$_w : S \cap [\phi] \neq \emptyset\} \neq \emptyset \qquad (\text{LA}_1)$$

Note that, if there are only finitely many spheres around w, then the limit assumption automatically holds. It may not be obvious why the existential quantification over spheres appearing in clause b) entails that ψ holds in the smallest sphere in which ϕ holds. This is exercise 3.

A diagrammatic representation of these two cases is given by Figures 1 and 2.

Remark 1 (Sub-clause a)**.** *The account renders all counterfactuals with impossible antecedents true. It is a straightforward matter to*

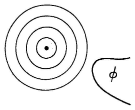

Figure 1. Vacuous truth

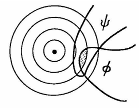

Figure 2. Non-vacuous truth

rephrase the truth-conditions for $\Box\!\!\rightarrow$ in such a way that all counterfactuals with impossible antecedents are trivially false. The choice between the two evaluation rules is merely a matter of convenience.

Remark 2 (Sub-clause b)**.** *This sub-clause implements Ramsey's test. The notion of possible world is the ontological analogue of the notion of stock of beliefs. The move to the closest/most similar world(s) correspond to a minimal adjustment of the agent's stock of beliefs to let the antecedent in.*

Remark 3 (Sub-clause b, ct'd)**.** *The closest* antecedent-*world need not be unique. (This is one of the points of disagreements between Lewis and Stalnaker.) One may motivate the need to allow for multiple closest* antecedent-*worlds by means of the following example:*

(5) *If Bizet and Verdi had been compatriots, they would have been either French or Italian*

The notions of validity, semantic consequence and satisfiability in (a class of) models are defined as usual.

Proposition 1. *The evaluation rules for $\Diamond\!\!\rightarrow$, $\Box$ and $\Diamond$ are as follows:*

- $M, w \vDash \phi \Diamond\!\!\rightarrow \psi$ *iff*
  a) $\exists S \in \$_w \exists w' \in S \ M, w' \vDash \phi$, *and*
  b) $\forall S \in \$_w((\exists w' \in S \ M, w' \vDash \phi) \Rightarrow (\exists w'' \in S \ M, w'' \vDash \phi \wedge \psi))$
- $M, w \vDash \Box\phi$ *iff* $\forall w' \in \cup\$_i : M, w' \vDash \phi$
- $M, w \vDash \Diamond\phi$ *iff* $\exists w' \in \cup\$_i : M, w' \vDash \phi$

Exercise 3. *Explain why, given the limit assumption, the existential quantifier over spheres appearing in clause b) entails that ψ holds in the smallest sphere in which ϕ holds.*

Exercise 4. *Explain the two ways that might arise for a counterfactual $\phi \Box\!\!\rightarrow \psi$ to be false (Hint: it all depends on whether or not $\phi \Box\!\!\rightarrow \neg\psi$ also holds.)*

Exercise 5. *Let $\Box\!\!\rightarrow'$ be like $\Box\!\!\rightarrow$ except they take opposite truth-value when the antecedent is 'impossible'. That is, the evaluation rule for the former is obtained from that for the latter, by just leaving a) out. Show that the two operators are inter-definable.*

2.3 Comparing $\Box\!\!\rightarrow$ with $\rightarrow$ / $-\!\!3$

One can try to appreciate how $\Box\!\!\rightarrow$ compares with material implication $\rightarrow$ or strict implication $-\!\!3$ (*fish hook*),[2] by contrasting the laws that govern each. Here $\Box\!\!\rightarrow$ appear to be weaker than $\rightarrow$ and $-\!\!3$, in the sense that the latter validates less laws than the formers.

Indeed, the distinctive feature of $\Box\!\!\rightarrow$ is that it does not satisfy the following three laws, which are characteristic of $\rightarrow$ and $-\!\!3$: strengthening of the antecedent; transitivity; and contraposition.

Below: some natural language examples showing that failure of these laws would indeed be expected of any reasonable semantics of counterfactual conditionals.

[2] Remember that $\phi -\!\!3\ \psi$ is definitionally equivalent with $\Box(\phi \rightarrow \psi)$.

Transitivity:

If R had gone to the party, S would have gone
If S had gone, then T would have gone
―――――――――――――――――――
If R had gone, then T would have gone

Strengthening of the antecedent:

If that match had been scratched, it would have lighted
―――――――――――――――――――
If that match had been wet and scratched, it would have lighted

Contraposition:

If R had gone, then S would have gone
―――――――――――――――――――
If S had not gone, then R would not have gone

Proposition 2. *The law of Strengthening of the Antecedent*

$$(\phi \Box\!\!\rightarrow \psi) \rightarrow ((\phi \wedge \phi') \Box\!\!\rightarrow \psi)$$

is not valid.

Proof. Let s, w and l stand for 'the match is scratched', 'the match is wet' and 'the match lights up'. Put $W = \{w_1, w_2, w_3\}$ and $\$_{w_1} = \{S_1, S_2, S_3\}$, where

$w_1 \models \neg s \wedge \neg w \wedge \neg l$	$S_1 = \{w_1\}$
$w_2 \models s \wedge \neg w \wedge l$	$S_2 = \{w_1, w_2\}$
$w_3 \models s \wedge w \wedge \neg l$	$S_3 = \{w_1, w_2, w_3\}$

Intuitively w_1 is the actual world, and w_2 is closer to w_1 than w_3. We have $s \Box\!\!\rightarrow l$ holds at w_1, but not $(s \wedge w) \Box\!\!\rightarrow l$, because l holds in the closest world in which s holds, but not in the closest world in which $s \wedge w$ holds. □

Exercise 6. *Show that the principle of transitivity and that of contraposition fail for $\Box\!\!\rightarrow$.*

2.4 Comparative similarity (CS) semantics

As mentioned, instead of working with a system of spheres, one may work with a two-place relation $\leq_w$ among worlds, regarded as the ordering of worlds in respect of their comparative similarity to w. For "$w_1 \leq_w w_2$", read "w_1 is at least as similar to w than w_2". The strict relation $<_w$ is defined by $w_1 <_w w_2$ whenever $w_2 \not\leq_w w_1$. R_w denotes the sets of worlds accessible from w.

Definition 4 (Comparative similarity system)**.** *Call $\{R_w, \leq_w\}_{w \in W}$ a comparative similarity system if the following constraints are met:*

- *$\leq_w$ is transitive and total (for all w', $w'' \in W$, either $w' \leq_w w''$ or $w'' \leq_w w'$)*
- *$w \in R_w$ (self-accessibility)*
- *$w \neq w' \Rightarrow w <_w w'$ (base world is always at the bottom)*
- *$w' \notin R_w \Rightarrow (\forall w'' \in W)\ (w'' \leq_w w')$ (inaccessible worlds are always at the top)*
- *$w' \in R_w$ & $w'' \notin R_w \Rightarrow w' <_w w''$ (accessible worlds are always below the inaccessible worlds)*

We can represent a CS ordering by a diagram with levels: the base world is on the lowest level; all points on a lower level are (strictly) more similar than those on a higher level; and all points on the same level being are equally similar to the base world. See Figure 3.

As with systems of spheres, the Hamming distance can be used to generate a CS ordering.

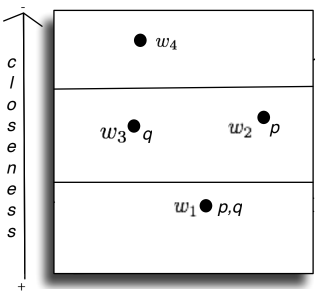

Figure 3. CS ordering

Exercise 7. *Show that totalness of $\leq_w$ implies reflexivity of $\leq_w$. Reflexivity and transitivity of $\leq_w$ translates each into a given different property on $<_w$. What are they?*

Exercise 8. *Explain how to define $\leq_w$ using the notion of Hamming distance.*

Definition 5 (CS model)**.** *A CS model is like a SOS model, except that \$ is replaced with a comparative similarity system $\{R_w, \leq_w\}_{w\in W}$ as defined supra. The evaluation rule for the would-conditional is rephrased thus:*

- $M, w \vDash \phi \mathrel{\Box\!\!\rightarrow} \psi$ *iff*
  a) $\neg\exists w' \in R_w\ M, w' \vDash \phi$, *or*
  b) $\exists w' \in R_w\ M, w' \vDash \phi$ *and*
  $\forall w''\ \ (w'' \leq_w w' \Rightarrow M, w'' \vDash \phi \rightarrow \psi)$

Intuitively: $\phi \mathrel{\Box\!\!\rightarrow} \psi$ holds at w if either ϕ is impossible or there is a ϕ-world accessible from w such that any world "below" it makes $\phi \rightarrow \psi$ true.

Example 1. *Consider the model M depicted in Figure 4. It must be understood that all the worlds are accessible from w. We have (for $i \in \{1, ..., 4\}$):*
$M, w \vDash p_i \mathrel{\Box\!\!\rightarrow} p_i$
$M, w \vDash \neg p_i \mathrel{\Box\!\!\rightarrow} p_{i+1}$
$M, w \vDash (p_i \wedge p_{i+1}) \rightarrow (p_i \mathrel{\Box\!\!\rightarrow} p_{i+1})$

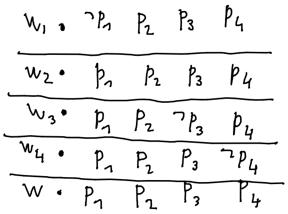

Figure 4. Example

Exercise 9. *Explain when a would-counterfactual is false at w. (Hint: look at what happens when the statement appearing at the right-hand side of the biconditional is negated.)*

Exercise 10. *A model is infinite if it has infinitely many worlds. Suppose the set of propositional letters, call it Prop, is infinite too. Give an infinite model similar to model M in example 1 in which the three $\Box\!\!\rightarrow$-statements mentioned in this example remain true at w, for all p_i in Prop.*

Proposition 3. *The evaluation rules for $\Diamond\!\!\rightarrow$, $\Box$ and $\Diamond$ are as follows:*

- $M, w \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \psi$ *iff*
  a) $\exists w' \in R_w\ M, w' \vDash \phi$, *and*
  b) $\forall w' \in R_w$(*if* $M, w' \vDash \phi$ *then*
  $\exists w'' \in R_w(w'' \leq_w w'\ \&\ M, w'' \vDash \phi \wedge \neg\psi))$
- $M, w \vDash \Box\phi$ *iff* $\forall w' \in R_w : M, w' \vDash \phi$
- $M, w \vDash \Diamond\phi$ *iff* $\exists lw' \in R_w : M, w' \vDash \phi$

A more user-friendly evaluation rule is available. Given a set X, let $\min_{\leq_w}(X)$ be the subset of those elements of X that are minimal under the relation $\leq_w$, viz.

$$\min_{\leq_w}(X) = \{x \in X : (\forall y \in X)(y \leq_w x \Rightarrow x \leq_w y)\}$$

The limit assumption mentioned above takes the form

$$[\phi] \cap R_w \neq \emptyset \Rightarrow \min_{\leq_w}([\phi] \cap R_w) \neq \emptyset \qquad (\mathrm{LA}_2)$$

Intuitively, (LA) rules out infinite sequences of closer and closer ϕ-worlds. Given (LA), the evaluation rule for the would-conditional may be rephrased thus:

- $M, w \vDash \phi \mathrel{\Box\!\!\rightarrow} \psi$ iff
  a) $\neg\exists w' \in R_w\ M, w' \vDash \phi$, or
  b) $\min_{\leq_w}([\phi] \cap R_w) \subseteq [\psi]$

Intuitively: $\phi \mathrel{\Box\!\!\rightarrow} \psi$ holds at w if either ϕ is impossible or ψ holds in every world that is minimal under the relation $\leq_w$ in the set of all worlds that are both accessible from w and satisfy ϕ.

Exercise 11. *Show that the following formula is valid under the CS semantics:*

$$(\phi \mathrel{\Box\!\!\rightarrow} \psi) \wedge (\phi \mathrel{\Box\!\!\rightarrow} \psi') \rightarrow (\phi \mathrel{\Box\!\!\rightarrow} (\psi \wedge \psi'))$$

Exercise 12. *Give a counter-example to the laws of transitivity, strengthening of the antecedent and contraposition under the CS semantics.*

2.5 Link with SOS models

The above two semantics are inter-derivable.

Call two models (with the same set of possible worlds) equivalent if a world satisfies exactly the same formulae in both models.

Theorem 1. *For every SOS model, there is an equivalent CS model.*

Proof. Let $M = (W, \$, V)$ a SOS model. The CS model derived from it is $M' = (W, \{R_w, \leq_w\}_{w\in W}, V)$, where, for each $w \in W$, R_w and $\leq_w$ are obtained thus:

- $R_w = \cup\$_w$
- $w_1 \leq_w w_2$ iff: $(\forall S \in \$_w)\ (w_2 \in S \Rightarrow w_1 \in S)$

It is a straightforward matter to show that $\{R_w, \leq_w\}_{w\in W}$ is a comparative similarity system. To show that the models are equivalent amounts to showing that a possible world satisfies exactly the same would-conditionals in both models. Details are omitted. □

Theorem 2. *For every CS model, there is an equivalent SOS model.*

Proof. Let $M = (W, \{R_w, \leq_w\}_{w \in W}, V)$ be a CS model. The SOS model derived from it is $M' = (W, \$, V)$, where for each w, $\$_w$ is $\{w' : w' <_w w''\} : w' \in R_w\}$. □

Exercise 13. *Fill in missing details in the proofs of Theorems 1 and 2.*

2.6 System VC

Definition 6. *The proof system* **VC** *is axiomatized using the following axioms and rules:*

R1	Modus-ponens for $\rightarrow$
R2	From $\wedge_{i=1}^{n} \psi_i \rightarrow \xi$ infer $\wedge_{i=1}^{n} (\phi \,\Box\!\!\rightarrow \psi_i) \rightarrow (\phi \,\Box\!\!\rightarrow \xi)$
R3	Interchange of logically equivalents
A1	Axioms of propositional logic
A2	$\phi \,\Box\!\!\rightarrow \phi$
A3	$(\neg\phi \,\Box\!\!\rightarrow \phi) \rightarrow (\psi \,\Box\!\!\rightarrow \phi)$
A4	$(\phi \,\Box\!\!\rightarrow \neg\psi) \vee (((\phi \wedge \psi) \,\Box\!\!\rightarrow \xi) \leftrightarrow (\phi \,\Box\!\!\rightarrow (\psi \rightarrow \xi)))$
A5	$(\phi \,\Box\!\!\rightarrow \psi) \rightarrow (\phi \rightarrow \psi)$
A6	$(\phi \wedge \psi) \rightarrow (\phi \,\Box\!\!\rightarrow \psi)$

The notions of proof, syntactical consequence and consistency are defined as usual.

Below: an example of proof in **VC**.

1. $\vdash \psi \wedge \psi' \rightarrow \psi$ (PL)
2. $\vdash (\phi \,\Box\!\!\rightarrow (\psi \wedge \psi')) \rightarrow (\phi \,\Box\!\!\rightarrow \psi)$ (1, R2)
3. $\vdash \psi \wedge \psi' \rightarrow \psi'$ (PL)
4. $\vdash (\phi \,\Box\!\!\rightarrow (\psi \wedge \psi')) \rightarrow (\phi \,\Box\!\!\rightarrow \psi')$ (3, R2)
5. $\vdash (\phi \,\Box\!\!\rightarrow (\psi \wedge \psi')) \rightarrow (\phi \,\Box\!\!\rightarrow \psi) \wedge (\phi \,\Box\!\!\rightarrow \psi')$ (2,4, PL)

Remark 4. *In* **VC**, $\Box$ *satisfies the axioms and rules of the modal system* **T**.

Theorem 3. **VC** *is sound with respect to i) the class of SOS models and ii) the class of CS models.*

Proof. By showing that the axioms are valid and the rules preserve validity. □

Theorem 4. **VC** *is complete with respect to i) the class of SOS models and ii) the class of CS models.*

Proof. Using the method of canonical models. □

Remark 5. **VC** *is also sound and complete with respect to the class of SOS (resp. CS) models in which the limit assumption is met.*

Theorem 5. **VC** *has the finite model property w.r.t. the two semantics described above, and thus the theoremhood problem in* **VC** *is decidable.*

Proof. Using the filtration method. □

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