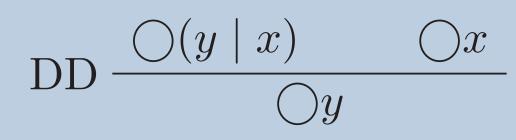
# Aggregative Deontic Detachment for Normative Reasoning

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#### The problem

Deontic Detachment



Counterexample

- a. You ought to exercise hard
- If you exercise hard, you ought to eat b. heartily
- ?\*You ought to eat heartily C.

Broome: "What, if you do not take exercise?" [1]

# Standard I/O system

**Definition 1** (Simple-minded, [5]).  $x \in out(N, a)$  iff  $x \in Cn(N(Cn(a)))$ , where  $Cn(X) = \{y : X \vdash y\}, and N(X) = \{y : (b, y) \in N, b \in X\}.$ 

Cf. Boghossian: modus-ponent is constitutive of the possession of the notion of conditional.

## **Removing W**

 $N[X] = \{x : x \Vdash \bigwedge_{i=1}^{n} x_i\}, \text{ where } N(X) = \{x_1, \dots, x_n\}. N \text{ is required to be finite.}$ 

**Definition 2** (Semantics).  $x \in \mathcal{O}^*(N, a)$  iff  $\exists M \subseteq N$  s.t.  $M(Cn(a)) \neq \emptyset$  and  $x \in M[Cn(a)]$ 

Define  $\mathcal{O}^{\star}(N) = \{(a, x) : x \in \mathcal{O}^{\star}(N, a)\}.$ 



This counterexample (and others, cf. [3, 4]) suggests an alternative (call it **aggregative**) form of detachment:

$$ADD \frac{\bigcirc (y \mid x) \qquad \bigcirc x}{\bigcirc (x \land y)}$$

This form of detachment has been overlooked in the literature.

#### Question

• Is there any interesting system supporting ADD, but not DD?

Accepting ADD, but not DD, implies rejecting W (Weakening)

$$W \frac{\bigcirc (x \mid a) \qquad x \vdash y}{\bigcirc (y \mid a)} \qquad ADD + W \to DD$$

#### Tasks

- 2-step semantics
  - remove W from standard systems
  - add ADD

**Definition 3** (Proof system).  $(a, x) \in \mathcal{D}^*(N)$  iff there is a derivation of (a, x) from N using the rules  $\{SI, EQ, AND\}.$ 

$$SI \frac{(a,x) \quad b \vdash a}{(b,x)} \qquad EQ \frac{(a,x) \quad x \dashv y}{(a,y)} \qquad AND \frac{(a,x) \quad (a,y)}{(a,x \land y)}$$

 $\mathcal{D}^{\star}(N,a) = \{ x : (a,x) \in \mathcal{D}^{\star}(N) \}.$ 

**Theorem 1** (Soundness and completeness).  $\mathcal{O}^*(N, a) = \mathcal{D}^*(N, a)$ 

*Proof.* See [7].

## Adding ADD

**Definition 4** (Semantics).  $x \in \mathcal{O}(N, a)$  iff  $\exists M \subseteq N$  s.t.  $M(Cn(a)) \neq \emptyset$  and  $x \in M[B]$  for all B with  $a \in B = Cn(B) \supseteq M[B]$ . Such a M is called an a-witness for x.

**Definition 5** (Proof system).  $(a, x) \in \mathcal{D}(N)$  iff there is a derivation of (a, x) from N using the rules  $\{SI, EQ, ACT\}.$ 

> ACT (a, x) $(a \wedge x, y)$

 $(a, x \wedge y)$ 

• Sound and complete axiomatization

## Our approach

In our work, we use so-called **input/output** (I/O) logic [5, 6]. The meaning of deontic concepts in given in terms of a set of procedures yielding outputs for inputs.

In I/O logic, a conditional obligation is represented as a pair (a, x) of boolean formulae, where a and x are the body (antecedent) and the head (consequent), respectively.

A normative system N is a set of such pairs. Below: our main construct

 $x \in O(N, a)$ 

Intuitively: given input a (state of affairs), x(obligation) is in the output under norms N. Equivalent notation:  $(a, x) \in O(N)$ .

ACT yields ADD as a special case (a is  $\top$ ).

**Theorem 2** (Soundness and completeness).  $\mathcal{O}(N, a) = \mathcal{D}(N, a)$ 

*Proof.* See [7].

#### Properties

**Property 1** (Bridge law).  $out'(N) = Cn(\mathcal{O}^*(N))$ , where out' is the standard "reusable" I/O operation [5]. (out' extends out to iterations of successive detachments.)

**Property 2** (Closure).  $\mathcal{O}^{\star}$  is a closure operator, viz

 $(x,y) \in N \Rightarrow y \in \mathcal{O}^{\star}(N,x)$ (1) $\mathcal{O}^{\star}(N) \subseteq \mathcal{O}^{\star}(N \cup M)$ (2) $M \subseteq \mathcal{O}^{\star}(N) \Rightarrow \mathcal{O}^{\star}(N) = \mathcal{O}^{\star}(N \cup M)$ (3)

(1), (2) and (3) express a principle of factual detachment, norm monotony, and norm induction, respectively.

**Property 3** (Violation detection).  $x \in \mathcal{O}^*(N, a) \Rightarrow x \in \mathcal{O}^*(N, a \land \neg x)$ .

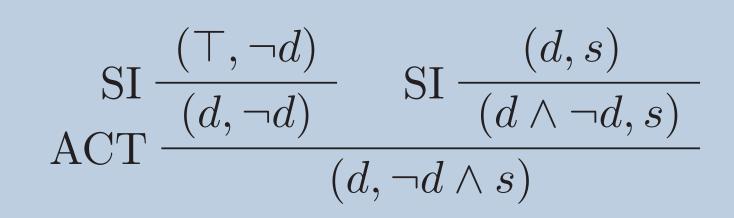
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- H. Prakken and M. Sergot, Contrary-to-duty obliga-2 tions, Studia Logica (1996)
- S. O. Hansson, Situationist deontic logic, Journal of Philosophical Logic (1997)
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- [6] L. van der Torre and X. Parent, Input/output logics, D. Gabbay & al. (eds), Handbook of Deontic Logic and Normative Systems (2013)
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Intuitively: in a violation context, obligations do not 'drown'. (This is a departure from nonmonotonic logics, which reject SI. Exceptions and violations should not be conflated.)

## The way forward

Pragmatic oddity [6]



#### in a cottage d: there is a dog s: there is a warning sign

**Definition 6** (Backtesting).  $x \in \mathcal{O}'(N, a)$  iff:  $a \vdash \wedge b$  with  $x \in \mathcal{O}(N, \wedge b)$  and  $\wedge b \cup \{x\} \not\vdash \bot$ .

Intuitively: go back in time, and check if x was obligatory before the violation occurred.